

ORIGINAL RESEARCH ARTICLE

Eur J Med Transl Biosci. 2026;1(1):26–38 · Open Access · CC BY 4.0 · Selected from ICMTB, peer reviewed by members of the editorial board

SINC FUNCTIONS FOR K-SPACE AND Z-SPACE ADAPTIVE FILTERING OF MAGNETIC RESONANCE IMAGING OF THE HUMAN BRAIN

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Abstract

Sampling at the Radon image grid determines sinusoidal waves that embed low-pass filtering characteristics. This paper describes and evaluates a newly proposed frequency domain adaptive filter by the name of Sinc Filter (SF). The Sinc Filter consists of an image whose pixels are the numerical values of the Sinc function computed at the Radon image space coordinates. The Sinc filter has two configurations. The first configuration is determined by numerical values of bandwidth and sampling rate that obey the sampling theorem. The second configuration is determined by the adaptive bandwidth and the value of the sampling rate. The pixel-local scaled k-space is the measure of the adaptive bandwidth, and it consists of the normalized k-space magnitude of the departing image. The Sinc Filter is then Fourier transformed or Z-transformed to obtain a frequency domain filter. Image smoothing is determined through: 1. The Fourier convolution theorem, which uses the frequency domain called k-space, and 2. The extension of the Fourier convolution theorem to Z-space, which uses the frequency domain called Z-space. For the two configurations, SF performance is compared to the performance of six frequency domain filters. Three filters make use of the k-space, and three more make use of the Z-space. The filters use Bessel, Butterworth, and Chebyshev based computed transfer functions. The Sinc Filter can smooth the images. The novelty of this paper is the adaptive frequency domain filtering procedure.

Keywords:

Fourier convolution theorem, Sampling theorem, Sinc Filter, Pixel-local scaled k-space, Smoothing, High-pass filtering.

1. Introduction

This research introduces an adaptive frequency domain filter called a Sinc Filter. Adaptive, in this study, refers to the pixel-by-pixel change of the Sinc Filter bandwidth through the pixel-local scaled k-space, rather than adaptive filtering in the traditional signal-processing sense. The Sinc Filter identifies its essential qualities by four equivalences [1]. 1. The signal processed by the Sinc Filter is a two-dimensional Magnetic Resonance Imaging (MRI) image. 2. The structure defining the relation between the input image and the filtered image is the Sinc Filter, which is conceptually equivalent to the transfer function. 3. The parameters within the structure are the Sinc coefficients, and they determine the pixel brightness of the Sinc Filter. 4. The algorithm is the step-by-step set of instructions for the calculation of the sampling points in the Radon coordinate system [2]. Overall, the elaboration on the four equivalences is possible through the knowledge reported in Williamson (1999) [1]. The Sinc Filter can be non-adaptive by not embedding

the frequency content of the MRI image, and conversely, adaptive by embedding the frequency content of the MRI image. Hence, the Sinc Filter has two configurations. The first one uses the values of bandwidth and sampling rate that comply with the sampling theorem [3]. The second one uses the estimate of bandwidth and a given value of the sampling rate. The estimate of the bandwidth is the pixel-local scaled k-space [4].

2. Materials and Methods

Radon-based sampling done in this research is not the same as that sampling done when the objective is the calculation of the sinogram of the image. In the *Radon-based coordinate sampling grid used in this research*, the grid point is defined by a pair of coordinates $(x_{(z, k)}, y_{(z, k)})$ as per equations (1), (2), (3) [5].

$$\begin{pmatrix} x_{(z, k)} \\ y_{(z, k)} \end{pmatrix} = \begin{pmatrix} \cos(\alpha_k) & \sin(\alpha_k) \\ \sin(\alpha_k) & -\cos(\alpha_k) \end{pmatrix} \cdot \begin{pmatrix} s_k \\ z_k \end{pmatrix} \quad (1)$$

Which implies that

$$x_{(z, k)} = s_k \cdot \cos(\alpha_k) + z_k \cdot \sin(\alpha_k) \quad (2)$$

$$y_{(z, k)} = s_k \cdot \sin(\alpha_k) - z_k \cdot \cos(\alpha_k) \quad (3)$$

$$s_k = \frac{x_{(z, k)} - z_k \sin(\alpha_k)}{\cos(\alpha_k)} \quad (4)$$

$$z_k = \frac{s_k \sin(\alpha_k) - y_{(z, k)}}{\cos(\alpha_k)} \quad (5)$$

The *Radon-based coordinate system* is used in this research to calculate the Sinc Filter by making use of the procedure outlined in this section. Equation (6) introduces the normalized angle α_k between the X axis of the XY Cartesian coordinate system and the *Radon-based coordinate system* [2].

$$\alpha_k = \frac{[2\pi \cdot i \cdot j] \cdot k}{[N \cdot N_x \cdot N_y]}; \text{ with } k = 1, 2 \dots N \quad (6)$$

In Eq. (6), $i = 0, 1 \dots N_y - 1$; $j = 0, 1 \dots N_x - 1$; N_x is the number of image pixels along the X direction; and N_y is the number of image pixels along the Y direction. N is the number of times the cycles of iterations repeat themselves as they are set by the counter k . The angle α_k changes for $i = 0, 1 \dots N_y - 1$; $j = 0, 1 \dots N_x - 1$, and then it changes again when $k = 1, 2 \dots N$. The angle $\alpha_k \in [0, 2\pi]$ determines the number of sinusoidal-like profiles that control the filtering properties of the Sinc Filter. In this paper, $N = 1$ or $N = 5$. The cycles of iterations set by the counters i and j are controlled by two 'for loops'. α_k is calculated $N_x \cdot N_y$ times to sample the whole Sinc Filter while covering all directions in the sampling grid. The terms in equations (2) and (3) are calculated by Eq. (9) and Eq. (12) [2].

$$dx = i - \frac{N_y}{2}; \text{ with } i = 0, 1 \dots N_y - 1 \quad (7)$$

$$dy = j - \frac{N_x}{2}; \text{ with } j = 0, 1 \dots N_x - 1 \quad (8)$$

$$s_k = \frac{dx}{\cos(\alpha_k)} \quad (9)$$

$$dy_{(z, k)} = s_k \cdot \sin(\alpha_k) \quad (10)$$

$$dy_{(z, k)} = z_k \cdot \sin\left(\frac{\pi}{2} - \alpha_k\right) \quad (11)$$

$$z_k = \frac{dy_{(z, k)}}{\sin\left(\frac{\pi}{2} - \alpha_k\right)} \quad (12)$$

$$\sin\left(\frac{\pi}{2} - \alpha_k\right) = \cos(\alpha_k) \quad (13)$$

$$\cos\left(\frac{\pi}{2} - \alpha_k\right) = \sin(\alpha_k) \quad (14)$$

The validity condition for Eq. (6) is met when $k = 1, 2 \dots N$; because $\alpha_k \in [0, 2\pi]$. Also, when $k = 1$ and $N = 1$, $\alpha_1 = \frac{[2\pi \cdot i \cdot j]}{[N_x \cdot N_y]} \in [0, 2\pi]$. The geometrical meaning of $\alpha_1 = \frac{[2\pi \cdot i \cdot j]}{[N_x \cdot N_y]}$ is that this angle, with changing counters i and j , makes it possible to sample all directions (s_k, z_k) . The angle α_1 is calculated $N_x \cdot N_y$ times to sample the whole Sinc Filter. The rationale of $\alpha_1 = \frac{[2\pi \cdot i \cdot j]}{[N_x \cdot N_y]}$ is to sample unevenly Radon-based coordinates $(x_{(z, k)}, y_{(z, k)})$. The justification for this rationale is to calculate the Sinc Filter, which is a task completely different versus the calculation of the Sinogram. To calculate the Sinc Filter, the sampling point in Radon-based coordinates $(x_{(z, k)}, y_{(z, k)})$ is given by equations (2), (3), (9), and (12), and it is used in the Whittaker-Shannon interpolation formula (see Eq. (15), Eq. (16), Eq. (20), and Eq. (21)). The Sinc functions used to calculate the first configuration of the Sinc Filter are formulated by Eq. (15) and Eq. (16), which are found in [2] (where they are named Equations (6) and (7)).

$$Sinc_{(x, k)} = Sinc \left\{ \pi \cdot \left[2.0 \cdot B \cdot x_{(z, k)} - \left(p - \frac{N_x}{2} \right) \cdot S \right] \right\}; k = 1, 2 \dots N \quad (15)$$

$$Sinc_{(y, k)} = Sinc \left\{ \pi \cdot \left[2.0 \cdot B \cdot y_{(z, k)} - \left(s - \frac{N_y}{2} \right) \cdot S \right] \right\}; k = 1, 2 \dots N \quad (16)$$

The couple of B (bandwidth) and S (sampling rate) comply with the sampling theorem [3]. Eq. (15) and Eq. (16) are the Sinc functions used in the *Radon-based coordinates sampling grid* coordinates $(x_{(z, k)}, y_{(z, k)})$; with $p = 0, 1 \dots N_x - 1$, and $s = 0, 1 \dots N_y - 1$. The Sinc function has filtering properties. The first configuration of the Sinc filter is Eq. (17), consistent with what is reported in [2].

$$I(i, j) = \sum_{s=1}^{N_x} \sum_{p=1}^{N_y} Sinc_{(x, k)} \cdot Sinc_{(y, k)} \quad (17)$$

Furthermore, the pixel-local scaled k-space $B^*(x, y)$ [4] is defined as

$$B^*(x, y) = \left(\frac{M(x, y) - \mu}{\sigma} \right) \cdot \left(\frac{1}{max - min} \right) \quad (18)$$

$M(x, y)$ is the k-space magnitude at each pixel location (x, y) , and μ, σ are its average and its standard deviation. The symbols *max* and *min* are the maximum and minimum values of $M(x, y)$. The pixel-local scaled k-space $B^*(x, y)$ is transformed using the function given in Eq. (19) [4].

$$B(x, y) = \frac{z_f}{[z_f + e^{-B^*(x,y)}]} \quad (19)$$

Where $z_f \in [0.1, 1.0]$, and $ARSinc_x$ and $ARSinc_y$ in Eq. (20) and Eq. (21) are the adaptive Sinc functions. The Sinc functions used to calculate the second configuration of the Sinc Filter are formulated by Eq. (20) and Eq. (21). The scientific novelty of this paper is the second configuration of the Sinc Filter, which is, the adaptive frequency filtering methodology based on the use of Radon-based sampling and the pixel-local scaled k-space. Indeed, this paper describes an adaptive frequency domain filtering method that uses Radon-based sampling and Fourier/Z convolution theorems. The image is filtered in k-space (or Z-space) by means of a transfer function sampled in Radon-based coordinates. The Radon-based coordinates $(x_{(z, k)}, y_{(z, k)})$ are used alongside with the pixel-local scaled k-space $B(x, y)$ by Eqs. (20), (21), and (22).

$$ARSinc_x = Sinc \left\{ \pi \cdot \left[2.0 \cdot B(x, y) \cdot x_{(z, k)} - S \cdot B(x, y) \cdot \left(p - \frac{N_x}{2} \right) \right] \right\} \quad (20)$$

$$ARSinc_y = Sinc \left\{ \pi \cdot \left[2.0 \cdot B(x, y) \cdot y_{(z, k)} - S \cdot B(x, y) \cdot \left(s - \frac{N_y}{2} \right) \right] \right\} \quad (21)$$

The second configuration of the Sinc Filter is calculated using Eq. (22).

$$I(i, j) = \sum_{s=1}^{N_x} \sum_{p=1}^{N_y} ARSinc_x \cdot ARSinc_y \quad (22)$$

Note that Eq. (20) and Eq. (21) in this paper differ from equations (6), (7) found in [2]. In equations (6), (7) in [2] B is the bandwidth. In Eq. (20) and Eq. (21) in this paper, the term $B(x, y)$ is the adaptive bandwidth. In this paper, Equations (15), (16) are the same as Equations (6) and (7) in [2] and are used to calculate the first configuration of the Sinc Filter. Whereas Eq. (20) and Eq. (21) in this paper are used to calculate the second configuration of the Sinc Filter. Regardless of the configuration, the Sinc Filter is an *image space map* conceptually equivalent to the filters' transfer function, like Bessel, Butterworth, or Chebyshev. The Sinc Filter is the object of a one-dimensional Fourier transform (or Z-transform). This transform is used by the Fourier convolution theorem [6] (or its extension to Z-space) [7] to determine filtering in the frequency domain.

3. Results

3.1. Merge with Existing Methods

This section reports the merge of the Sinc Filter (SF) with six frequency domain filters. The filters used for the merge are Bessel in k-space and Z-space, Butterworth in k-space and Z-space, and Chebyshev in k-space and Z-space. Figures 1, 2, and 3 provide evidence of the effectiveness of the filtering procedures combining the use of the specific frequency domain filter with the SF. Figures 1, 2, and 3 display images calculated using the program by the name 'ImageReconstruction2025v1' (see top row in Table 1) hence using the second configuration of the Sinc Filter (the adaptive SF). Figure 1 shows the combination of the

Bessel filter and SF. Figure 2 shows the combination of Butterworth filter and SF. Figure 3 shows the combination of Chebyshev filter and the Sinc Filter. The figures show two k-space filtering procedures (see the top row and the third row from the top), and two Z-space filtering procedures (see the second row from the top and the bottom row). Table 2 is composed of three sub-tables. The sub-table positioned at the top is illustrative of Figure 1. The sub-table positioned in the middle is illustrative of Figure 2. The sub-table positioned at the bottom is illustrative of Figure 3. Bessel, Butterworth, and Chebyshev filters use the values of cut-off frequency, number of poles, bandwidth and sampling frequency reported in each sub-table. The Z-transform makes use of the complex number reported in the table. The second configuration of the Sinc Filter uses the value of sampling frequency reported in the bottom row of each of the three sub-tables.

Table 1. Filtering procedures and experimental settings. First column from the left: calculation of k-space or Z-space of TF using Bessel, Butterworth, or Chebyshev polynomials. In the second and third columns from the left, the table refers to the first and second configurations of the Sinc Filter, respectively. In fact, the Sinc Filter is calculated either: 1. non-adaptively by a couple of bandwidth and sampling rate values that are chosen to fulfil the sampling theorem (first configuration ON, second configuration OFF); or 2. adaptively by means of the pixel-local scaled k-space (first configuration OFF, second configuration ON). The rightmost column shows the names of the software programs that implement filtering procedures. Ten software programs aid the development of this research.

Calculation of the k-space or Z-space of the transfer function (TF). The TF is calculated using Bessel, Butterworth, or Chebyshev polynomials	Calculation of the k-space or Z-space of the Sinc Filter. No use of Adaptive Sampling (First Configuration)	Calculation of the k-space or Z-space of the Sinc Filter. Use of Adaptive Sampling (Second Configuration)	Row #	Program Name
ON	OFF	ON	1	ImageReconstruction2025v1
OFF	OFF	ON	2	ImageReconstruction2025v2
ON	ON	OFF	3	ImageReconstruction2024v5
OFF	ON	OFF	4	ImageReconstruction2024v6
ON	OFF	OFF	5	kSpaceBesselFilter2024
			6	kSpaceButterworthFilter2024
			7	kSpaceChebyshevFilter2024
			8	ZSpaceBesselFilter2024
			9	ZSpaceButterworthFilter2024
			10	ZSpaceChebyshevFilter2024

3.2. Comparison with Existing Methods

Figures 4 and 5 expand the presentation of the results showing the effect of the filtering procedures reported in each of the rows of Table 1. Figure 4, which is composed of four rows of images, presents results obtained by the four procedures reported in Table 1 in rows 1, 2, 3, and 4, respectively. The top row of Figure 4 makes use of the procedure visible in row 1 of Table 1; the second row from the top in Figure 4 refers to row 2 of Table 1, and so forth. Figure 5 displays images obtained by the procedures reported in Table 1 in rows 5 through 10. Each row of Table 1 describes the software program name that implements the filtering procedure.

Table 2. Filters' parameters for Figures 1, 2, and 3. The table is composed of sub-tables: the one on top for Figure 1, the one in the middle for Figure 2, and the one at the bottom for Figure 3. Each sub-table reports two cases processed using the Fourier convolution theorem (see (a) and (c)), and two more cases processed using the Fourier convolution theorem extension to Z-space (see (b) and (d)). The Z-transform uses the complex numbers shown in the table. Bessel, Butterworth, and Chebyshev filters use the cut-off frequency, number of poles, bandwidth, and sampling frequency located in the table. The second configuration of the Sinc Filter uses the sampling frequency as indicated in the table.

Filter	Bessel in k-space - case (a) in figure 1	Bessel in Z-space - case (b) in figure 1	Bessel in k-space - case (c) in figure 1	Bessel in Z-space - case (d) in figure 1
Parameters				
Complex Number	-	$(0.5 + i 0.5)$	-	$(0.3 + i 0.5)$
Cut-off F.	2.5	2.5	2.5	2.5
N. of Poles	5	2	5	5
Bandwidth	0.0013	0.0445	0.0445	0.0013
Sampling Frequency	0.13	0.145	0.445	0.013
Sampling Frequency of the Sinc Filter	1.5	1.5	2.3	2.3
Filter	Butterworth in k-space - case (a) in Figure 2	Butterworth in Z-space - case (b) in Figure 2	Butterworth in k-space - case (c) in Figure 2	Butterworth in Z-space - case (d) in Figure 2
Parameters				
Complex Number	-	$(0.5 + i 0.5)$	-	$(0.5 + i 0.5)$
Cut-off F.	2.5	2.5	2.5	2.5
N. of Poles	5	5	5	5
Bandwidth	0.0013	0.013	0.00445	0.00445
Sampling Frequency	0.13	0.13	0.445	0.445
Sampling Frequency of the Sinc Filter	1.5	1.5	2.3	2.3
Filter	Chebyshev in k-space - case (a) in Figure 3	Chebyshev in Z-space - case (b) in Figure 3	Chebyshev in k-space - case (c) in Figure 3	Chebyshev in Z-space - case (d) in Figure 3
Parameters				
Complex Number	-	$(0.3 + i 0.1)$	-	$(0.3 + i 0.1)$
Cut-off F.	2.5	2.5	2.55	2.55
N. of Poles	5	6	4	4
Bandwidth	0.0013	0.0013	0.45	0.45
Sampling Frequency	0.13	0.13	1.45	1.45
Sampling Frequency of the Sinc Filter	1.5	1.5	2.3	2.3

4. Discussion

It is worth recalling that the word adaptive filtering in this paper is not strictly related to the norm in the literature that relates to the design of time-domain and frequency-domain adaptive filters. The difference is that the adaptive feature of the algorithm presented in this paper aims to bring to the filter transfer function the changeability of the Sinc function.

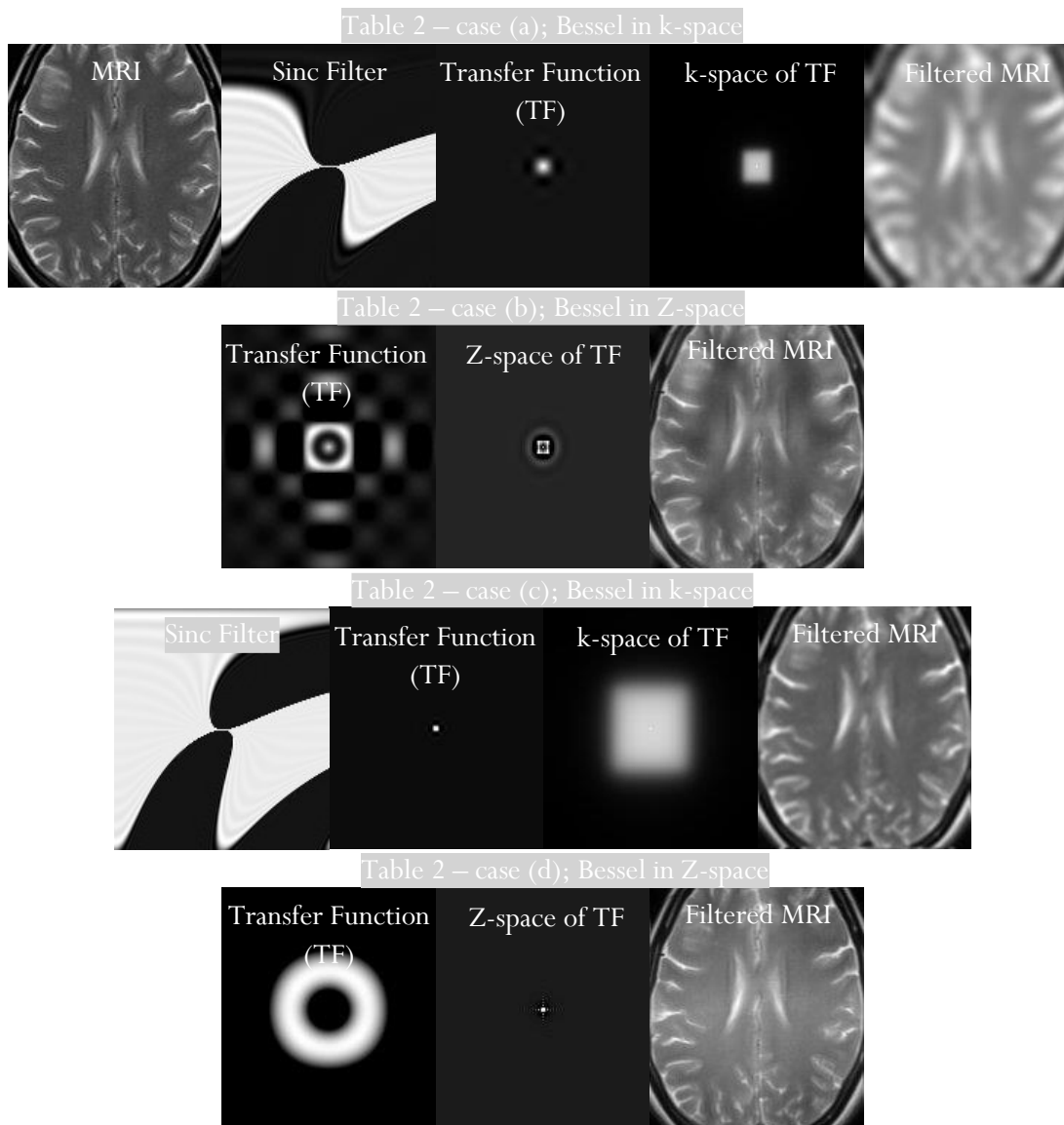


Figure 1. Evidence of effective combined use of the Sinc Filter and the Bessel filter. The figure shows the images of the filtering procedures. There are 4 procedures, 2 in k-space and 2 in Z-space. For each of them a transfer function is generated by making use of the Bessel polynomial. The Fourier convolution theorem reconstructs the MRI by inverse Fourier transform of the pointwise multiplication of the k-space of the Bessel filter, the k-space of SF, and the k-space of MRI. The extension of the Fourier convolution theorem to Z-space reconstructs the MRI by inverse Z-transform of the pointwise multiplication of the Bessel filter, the Z-space of SF, and the Z-space of MRI. Table 2 reports the parameters used in each of the filtering procedures.

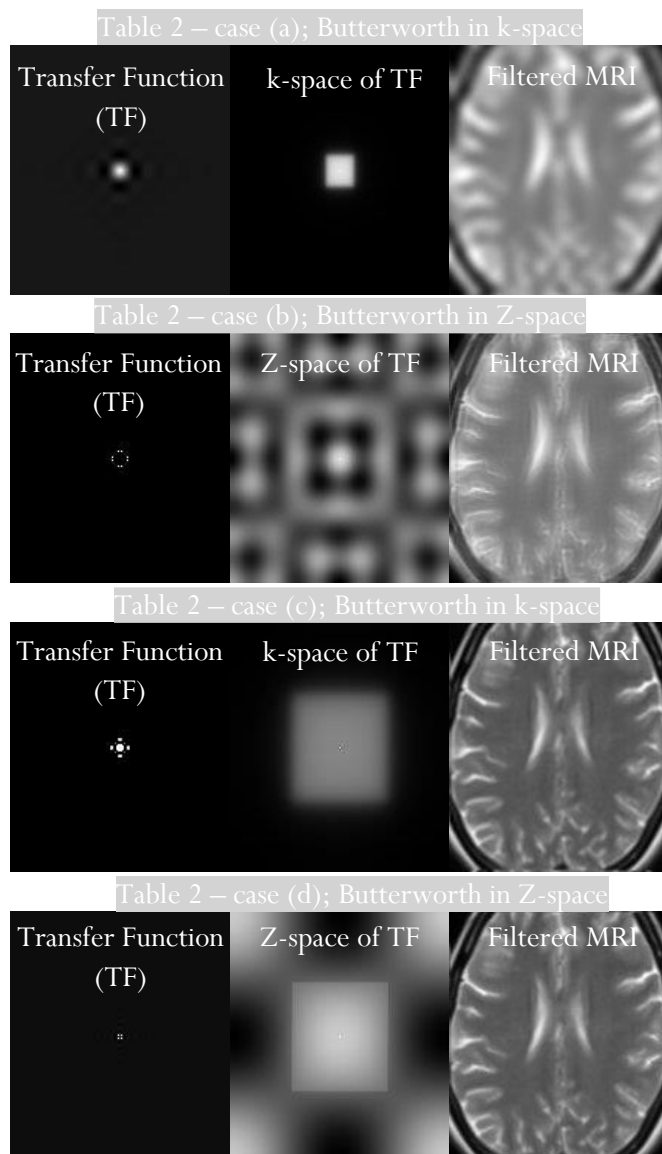


Figure 2. Evidence of effective combined use of the Sinc Filter and the Butterworth filter. Like in figure 1, there are 4 procedures, 2 in k-space and 2 in Z-space. The transfer function is generated using the Butterworth polynomials. The Fourier convolution theorem makes use of the k-space of the Butterworth filter, the k-space of SF, and the k-space of the MRI. The extension of the Fourier convolution theorem to Z-space makes use of the Z-space of the Butterworth filter, the Z-space of SF, and the Z-space of MRI. Table 2 provides the parameters used by the filters.

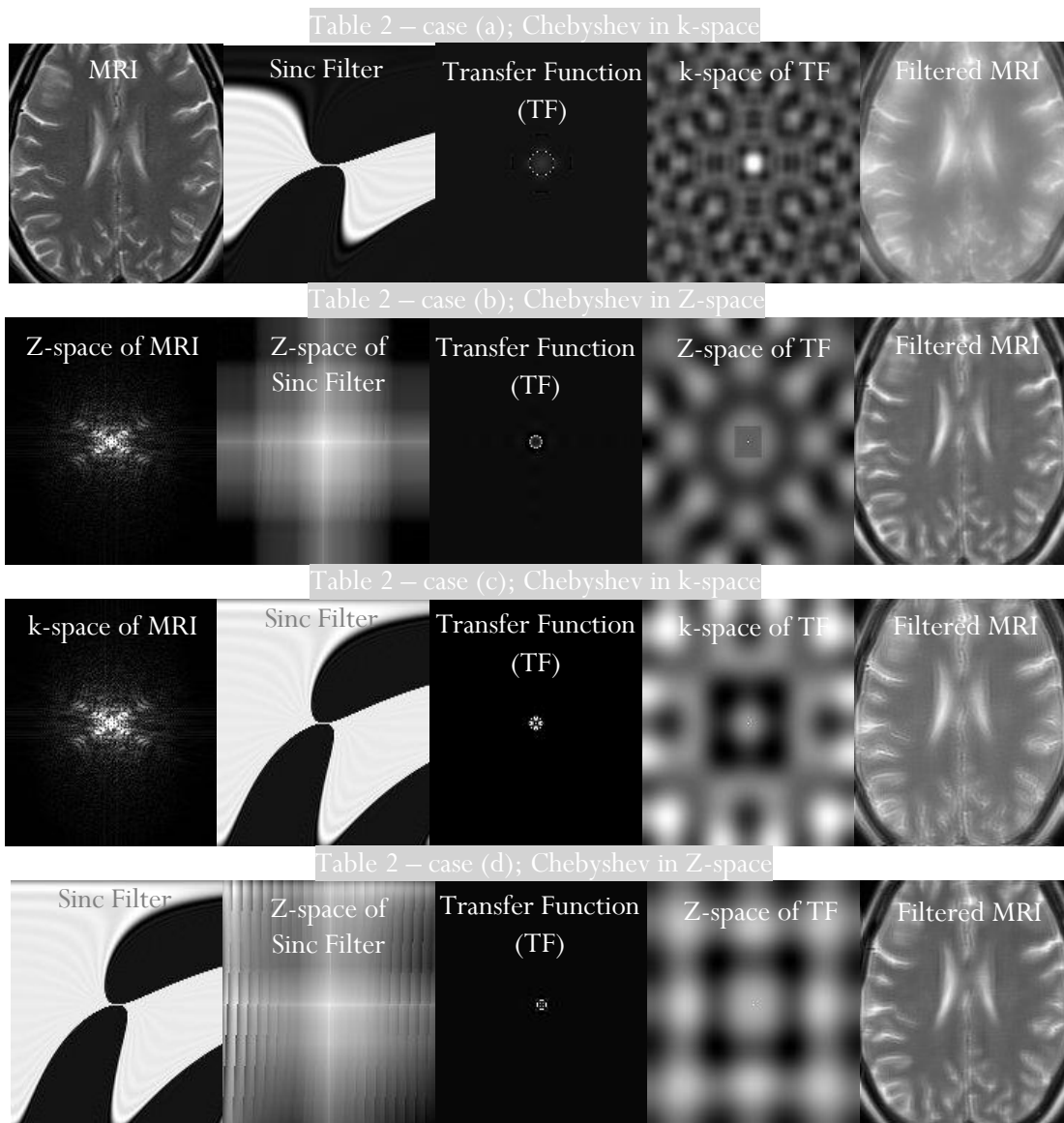


Figure 3. Evidence of effective combined use of the Sinc Filter and the Chebyshev filter. First and third row – The filtered MRI was obtained using the Fourier convolution theorem. Image reconstruction was obtained by inverse Fourier transform of the pointwise multiplication of the k-space of the Chebyshev filter, the k-space of the SF, and the k-space of the MRI. Second and fourth row – The filtered MRI was obtained using the extension of the Fourier convolution theorem to Z-space. Image reconstruction was obtained by inverse Z-transform of the pointwise multiplication of the Z-space of the Chebyshev Filter, the Z-space of the SF, and the Z-space of the MRI. Table 2 reports the numerical values of the parameters involved in these computations.

Table 3. Filter parameters for figures 4 and 5. The Z-transform uses the complex number shown in the table. Bessel, Butterworth, and Chebyshev filters use the cut-off frequency, number of poles, bandwidth, and sampling frequency as indicated in the table. The first configuration of the SF (executed by ImageReconstruction2024v5) uses the sampling frequency and the value of the bandwidth in accordance with the Nyquist theorem (see equations (15) and (16)). The second configuration of the SF (executed by ImageReconstruction2025v1 and ImageReconstruction2025v2) uses the sampling frequency and the pixel-local scaled k-space (see the adaptive bandwidth in equations (20) and (21)). The two right-most columns connect to figures 4, 5, and table 1.

Filter Parameters	C ^a	CF ^b	N ^c	B ^d	SF ^e	SF ^f	K-Z ^g	F ^h	Figs. ⁱ	Table 1 ^j
Program Name										
ImageReconstruction2025v1	(0.3 + i 0.1)	2.5	5	0.0287	0.287	0.87	Z	Bessel	Fig. 4, row 1	1
ImageReconstruction2025v2	-	-	-	-	-	0.87	k	-	Fig. 4, row 2	2
ImageReconstruction2024v5	(0.3 + i 0.1)	2.55	5	0.0287	0.287	-	Z	Butterworth	Fig. 4, row 3	3
ImageReconstruction2024v5	(0.3 + i 0.1)	2.55	4	0.0287	0.287	-	Z	Chebyshev	Fig. 4, row 3	3
ImageReconstruction2024v6	(0.3 + i 0.1)	-	-	-	-	-	Z	-	Fig. 4, row 4	4
kSpaceBesselFilter2024	-	2.7	5	0.0017	0.17	-	k	Bessel	Fig. 5, row 1	5
kSpaceButterworthFilter2024	-	2.7	3	0.0017	0.17	-	k	Butterworth	Fig. 5, row 2	6
kSpaceChebyshevFilter2024	-	2.55	6	0.0287	0.287	-	k	Chebyshev	Fig. 5, row 3	7
ZSpaceBesselFilter2024	(0.3 + i 0.1)	2.5	5	0.0287	0.287	-	Z	Bessel	Fig. 5, row 4	8
ZSpaceButterworthFilter2024	(0.3 + i 0.1)	2.7	3	0.0287	0.287	-	Z	Butterworth	Fig. 5, row 5	9
ZSpaceChebyshevFilter2024	(0.3 + i 0.1)	2.45	6	0.0287	0.287	-	Z	Chebyshev	Fig. 5, row 6	10

^a Complex Number. ^b Cut-off Frequency. ^c Number of Poles. ^d Bandwidth. ^e Sampling Frequency. ^f Sampling Frequency of the Sinc Filter. ^g k-Space / Z-space. ^h Filter. ⁱ Figs. 4, 5, row # from the top. ^j Table 1 row # from the top

In fact, the Sinc Filter (SF) in this paper is a sum of Sinc functions calculated on the uneven grid of the Radon two-dimensional domain. Moreover, this paper designs an adaptive filtering procedure in image processing through the reconstruction ability of the Fourier convolution theorem [6] and its extension to the Z-domain [7]. So, both in principle and in practice (implementation), this paper differs from the traditional theory on adaptive filters designed and used in signal processing.

5. Conclusions

The Sinc Filter can have two configurations. The first configuration uses a predetermined pair of values of the bandwidth and the sampling rate that comply with the Nyquist theorem. The second configuration uses the pixel-local scaled k-space as the adaptive bandwidth in combination with the value of the sampling rate. The SF is compared to six frequency domain filters. First, evidence of effective combined use of the Sinc Filter and the 6 frequency domain filters is given in figures 1, 2, and 3. Second, the comparison with the 6 frequency domain filters is qualitatively illustrated in figures 4 and 5. The main difference between the SF and the aforementioned 6 frequency domain filters is its smoothing effect. In the experiments presented in this paper, the principal qualitative effect observed while using the SF is

soft smoothing of the MRI images. Although not presented by the results on display in this paper, the Sinc Filter also displays edge-finding behaviour, and this happens when the SF applies high-pass filtering. This aspect will be the object of presentation in a later research paper. The limitation of the SF filter is the computational demand. The computational time of the programs running on a PC with 6.00 GB RAM and an i5 Intel 3.30 GHz processor is at most 7 mins and 5 sec. In conclusion, the main benefit of SF is to provide soft smoothing.

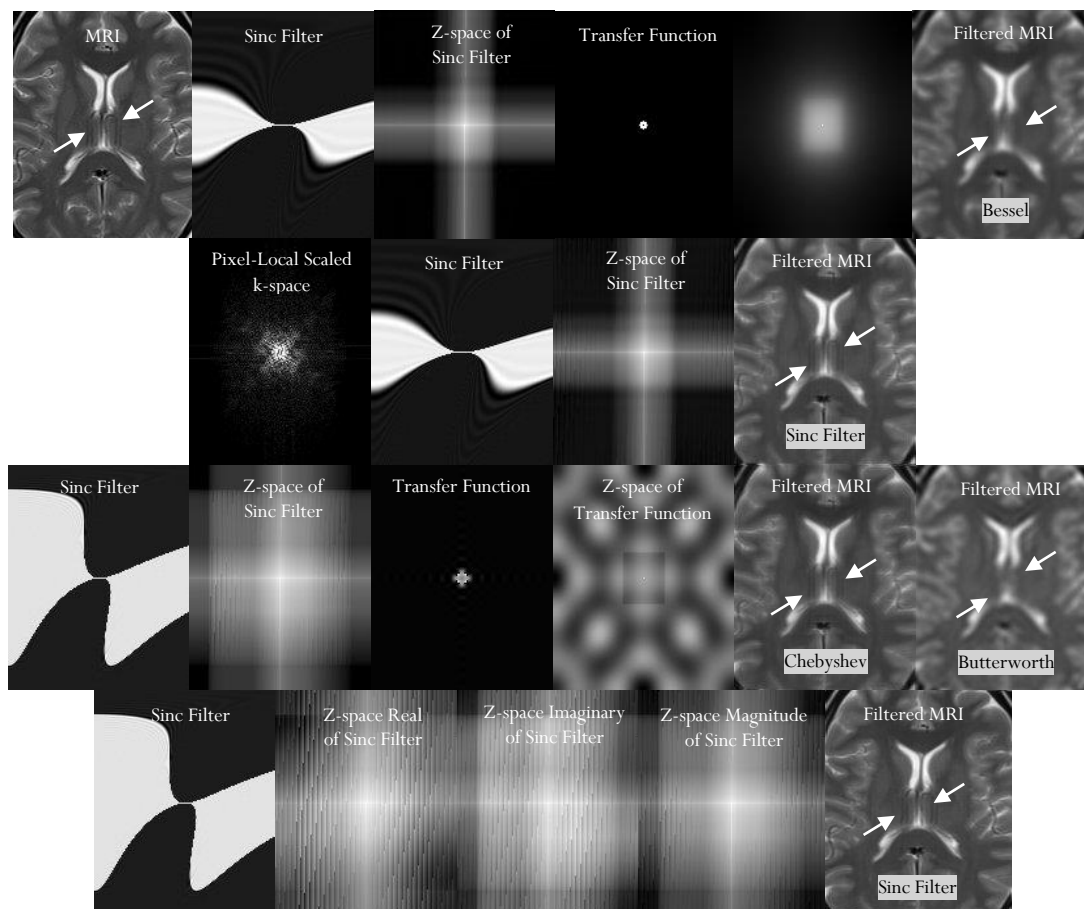


Figure 4. Images obtained by low-pass filtering procedures that use the Sinc Filter. The images have two references. One is table 1, which reports the filtering procedures, and the other one is table 3, which reports the parameters used for the calculations. There are four rows in this figure and each of them shows images calculated using the experimental setting reported in the first four rows of table 1 and the first five rows in table 3.

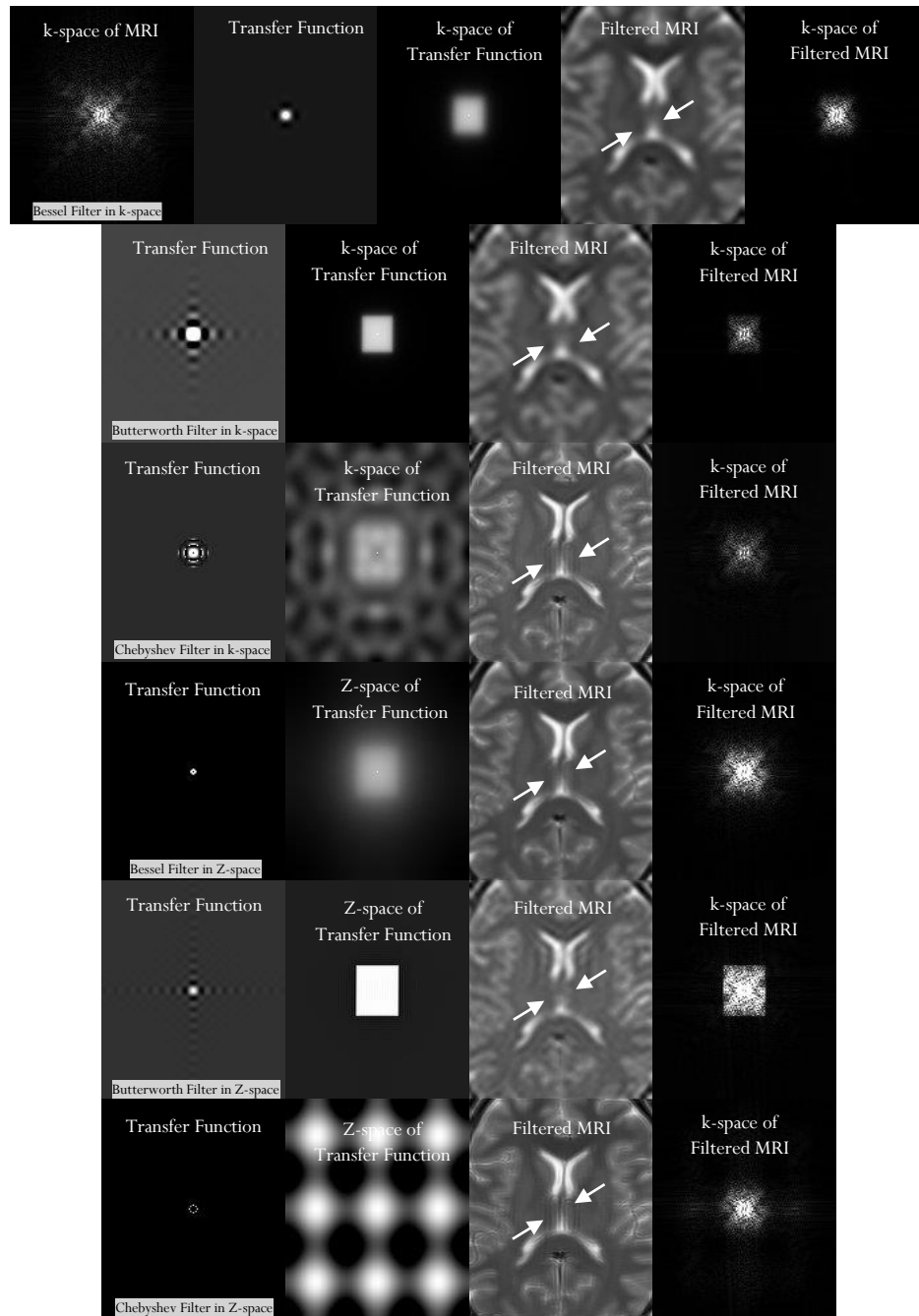


Figure 5. Filtering procedures that are listed in table 1 rows 5 through 10, and table 3 in rows 6 through 11. These 6 cases use 3 k-space filters and 3 Z-space filters. The transfer functions are calculated using Bessel, Butterworth, and Chebyshev polynomials. The parameters of the filters are reported in table 3. Note the arrows pointing to a detail in the MRI. The detail is filtered out in row 1 (see filtered image), row 2 (Butterworth), row 4, row 5, and it is preserved in rows 3 (Chebyshev), and 6.

Acknowledgements

The author expresses sincere gratitude for the MR images provided by Dr Dimitar Veljanovski and Dr Filip A. Risteski. The author is also grateful to Professor Ustijana Rechkoska Shikoska for the coordination of human resources. Dr Dimitar Veljanovski and Dr Filip A. Risteski are affiliated with the Department of Radiology at the General Hospital 8-mi Septemvri located on Boulevard 8th September in the city of Skopje, Republic of North Macedonia. Professor Ustijana Rechkoska Shikoska is affiliated with the University of Information Science and Technology (UIST) located in Partizanska BB in the city of Ohrid - Republic of North Macedonia. The MR images shown in this paper serve to evaluate the filtering algorithm studied in this research.

Ethics approval, consent to participate, and consent for publication: MRI acquisition and use of the MR image for research purposes were consented to by the subjects through the administration of written consent. The Department of Radiology, at the General Hospital 8-mi Septemvri located in Boulevard 8th September in the city of Skopje - Republic of North Macedonia, approved the MRI data acquisition protocol. This study follows the principles stated in the Declaration of Helsinki of 1964.

Funding: No funding supports this research. **Competing interests:** None. **Data availability:** The datasets generated and/or analyzed during the current study are available from the corresponding author on reasonable request. **Code availability:** Software is freely available at <https://github.com/cxc2728/SincFilter>. **Author's contribution:** conceptualization, software, data processing, writing, editing.

Declaration of Generative AI and AI-Assisted Technologies

The author declares that no generative AI or AI-assisted technologies supported the preparation of this manuscript.

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How to cite: Ciulla C. Sinc Functions for K-Space and Z-Space Adaptive Filtering of Magnetic Resonance Imaging of the Human Brain. Eur J Med Transl Biosci. 2026;1(1):26–38.